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Developing methods to improve sampling efficiency for automated soil mapping

by

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CONTENTS

ABSTRACT

The goal of this project was to develop methods to sample spatial variables, such as soil or crop properties, which are efficient and cost-effective despite the fact that we start with little or no information about the spatial variability of the variable. Commonly when we begin a survey we do not know in advance how intensively to sample, and so we run the risk of completing then survey then finding that we have substantially over-sampled and so wasted effort, or that we are not able to produce a reasonable map from the our data because they are too sparse.

We developed an approach to optimization of sampling for a single-phase geostatistical survey, showing how the combined uncertainty of our model of spatial variation (variogram), and predictions under spatial variation could be quantified and minimized, assuming some prior distribution of variogram parameters. We showed, using simulation, how this scheme successfully models prediction error variance, and how under different underlying kinds of spatial variation the resulting sample schemes achieve the dual goal of allowing adequate estimates of the variogram and disposing sample points from which to map the variable.

We then developed a fully adaptive sample scheme. Under this the sampling is divided into phases. In the initial phases (reconnaissance) our uncertainty about the required final sample intensity is reduced by sampling designed to yield maximum information, and at the end of each phase we are presented with a representation of this uncertainty which allows us to decide whether further reconnaissance sampling is justified or whether we should proceed to a final sampling phase before mapping. This final phase may be designed so as to allow for the existing observations, and so to save on total sampling effort. We demonstrated the value of this method by simulation.

We built a field system to implement these algorithms for mapping the water content of the soil with a sensor. This uses GPS to record locations, and to guide the user to selected sample sites. We applied the system in a survey of a field, and showed how it identified the relatively sparse sample effort needed to map the variable to a specified precision.

In conclusion, we have shown how substantial efficiencies in sampling are possible using appropriate algorithms. Some of the methods we have used, notably the spatial simulated annealing algorithm to select sample points, could be applied to simpler sampling problems that farmers and agronomists face.

SUMMARY

Background: the problem

Spatial variability of soil and other variables at within-field scales is recognized as both a challenge and an opportunity for farmers and their advisers (see, for example, the HGCA Research Strategy, 2004). It is also recognized that one of the biggest problems for management of within-field variation is obtaining adequate information on the variations of important variables at acceptable cost.

These issues have been tackled in previous projects funded by the HGCA. Lark et al. (1998, 2003) considered, among other problems, how ancillary variables, notably yield data, might be used to direct sampling more efficiently. One of their principal conclusions, based on a large set of fields across the arable landscape of Great Britain, was that

while all fields are variable, some fields are more variable than others.

This gnomic statement has significant implications. Most importantly it means that we cannot, with confidence, present farmers or agronomists with a sampling strategy that will be efficient in all fields. This conclusion is supported by the work of Oliver and Frogbrook (2004) who showed that, if soil properties are to be mapped with adequate precision, then the effort needed will differ between fields. Their results suggested that the standard practice of sampling on a 100-m grid was unlikely to resolve spatial variation adequately.

The farmer or agronomist is left in a quandary. It may be summarized thus:

> *I am told that sampling every 100m is unlikely to generate adequate information for spatially variable management, but I am not given an alternative intensity against which I can make judgements about the costs and benefits of this information. On the basis of what we are told a 50-m sample interval (four times more costly than the standard) might turn out to be woefully inadequate in some fields and wastefully generous in others. How, then, am I to decide?*

The research cited above provides some answers. Lark et al. (2003) presented an algorithm to make a prediction in absolute terms as to whether the scales and magnitudes of variation within a field are likely to justify attempts at precision management. They also showed how

yield maps could be used to target exploratory sampling of a field to identify which, if any, variables might be important enough to justify a detailed survey. The strategy that they developed was based on the principle of taking sampling in small steps, each designed to support the decision as to whether to take further steps as cost-effectively as possible. King et al. (2004) considered in more detail the potential of remote sensing technologies, notably EMI surveys, as a source of information for such decision making. Oliver and Frogbrook (2004) suggested that cheap data generated by these and similar technologies might be used to identify the spatial scales in a field that might reasonably be assumed to be reflected in important but costly to measure soil properties. From this might be inferred the spatial intensity needed for a soil survey. These approaches are admittedly rough and ready. The approach of Lark et al (2003) leaves open the problem of how intensively to sample once a rational decision has been made to do so, and the approach of Oliver & Frogbrook (2004) is based on a hypothesis that ancillary variables and key soil properties have sufficiently similar spatial variation. While the results presented are encouraging, the robustness of this hypothesis still needs testing across a wide range of conditions.

A proposed solution: adaptive spatial sampling

This report presents research in which we lay the foundations for a different approach to the problem. The work is particularly inspired by the increasing interest in sensor technologies that return immediate measurements of soil or crop variables. Some such technologies are well-established (soil penetrometers, soil moisture probes, leaf chlorophyll meters) while others are at earlier stages of development (notably sensors to measure chemical properties of the soil, such as pH, Viscarra Rossel & McBratney, 2000; or nitrate concentration, Miller et al., 2003). Some of these sensors might be deployed on tines and dragged through the soil during field operations, but others might be best deployed at sites by field workers, or may be in the future by autonomous vehicles such as those developed for certain field operations in row crops at Silsoe Research Institute (Hague et al., 2000). Such technologies may tip the balance of costs and benefits in favour of more intensive sampling, but they would still need to be deployed efficiently because of (i) the costs of sensor components, (ii) the cost of staff time or time of a costly vehicle and (iii) the need to generate timely information on many variables.

In this context we propose an adaptive approach to sampling. In short, rather than deploying all the sample effort in a single designed phase, we split it up into several phases (the total number of which may not be known in advance). In early phases we are concerned primarily to characterize the spatial variability of the variable of interest, in later phases we focus on sampling to support mapping of the variable by geostatistical interpolation. Any phase is designed using information from past phases (including the known values of the property at previously-sampled sites) and generates information used to decide how to sample next. We hypothesize that, by such adaptive sampling, it should be possible to arrive at an efficient final sampling scheme in the absence of any prior information on variability. However, information from ancillary sources, such as those proposed by Oliver & Frogbrook (2004) could be used to initiate the process, although if it turns out that the ancillary data is a poor guide in any specific case this will be recognized.

The proposed approach draws on various geostatistical studies. The earliest is that by McBratney et al. (1981) who showed how, given a variogram function that describes the spatial variability of a property, we may plan a sampling scheme to ensure that the property is mapped to some specified precision. This left open the problem of how to specify the variogram, given that this must be based on data, and how we should allow for uncertainty in the variogram estimates since these result in uncertainty both in the sample design and the estimates generated after sampling. We tackle these problems in this project. Another important precursor to our work is the study by Van Groenigen (1999). This showed how an optimization method, simulated annealing, can be used to generate a spatial sampling array that minimizes some objective function (such as the maximum expected error variance of a geostatistical prediction at any site in a field). Lark (2002) applied this to the problem of variogram estimation, showing how the best sample design for this purpose depends on the underlying variogram itself, and demonstrating the principle of adaptive approaches in a two-phase scheme where an estimate of the variogram from an initial sample on transects is used to choose the disposition of a further set of points to refine this estimate.

This sets the context for the work that we present here. We tackle the following questions.

1. **How can the uncertainty attendant on an estimated variogram model be described quantitatively?** This is necessary for our purpose if we are to allow for the uncertainty of variograms estimated in early phases of sampling when designing further phases. As is indicated in Chapter 1 (an abstract of a published paper), there are problems with the methods that have previously been proposed for this, and we tested and evaluated these as a necessary preamble to subsequent steps.

2. **Can we design sampling schemes for geostatistical mapping in which the overall uncertainty of predictions, due both to spatial variation and uncertainty in our variogram, are known and minimized?** In this way we deal with the limitation in

the approach of McBratney et al (1981) that the variogram is assumed known. However, the problem is a complex one since the uncertainty of a variogram, estimated from a particular sample scheme, depends on the (unknown) variogram itself. We address these problems in Chapter 2.

3. **How can the uncertainty about the specifications of a final sample grid, given a set of observations and their experimental variogram, be quantified, and can this be built into an adaptive sampling scheme for any variable of unknown variability?** These problems are addressed in Chapter 3 of this report.

4. **Can these schemes be incorporated into a working field system?** In Chapter 4 we describe briefly a field system, and illustrate its use in a reconnaissance survey to identify a sampling scheme for mapping soil water content.

Towards adaptive sampling: what we have done

1. **How can the uncertainty attendant on an estimated variogram model be described quantitatively?**
   We concluded that existing expressions for the uncertainty of variogram parameter estimates, which were largely untested before this study, could give misleading predictions of the error in non-linear parameters: i.e. the distance parameters which control the distance over which a variable shows spatial dependence. Since this has significant implications for problems like design of a survey grid, this put limitations on how far we could use these expressions in further work, as well as being an important finding for geostatistics in its own right. We used a Bayesian formulation of variogram uncertainty for further work.

2. **Can we design sampling schemes for geostatistical mapping in which the overall uncertainty of predictions, due both to spatial variation and uncertainty in our variogram, are known and minimized?**
   In this part of the project we addressed the problem of how to optimize a sampling scheme in one phase to support both estimation of the variogram and subsequent kriging. The critical issue here was to allow for how error in the specified variogram will propagate through to error in the kriging estimates. We derived an expression for the combined effects of spatial variation, and error in our model of this (variogram) on the error variance of kriged predictions, and tested these by simulations. Using spatial simulated annealing (Van Groenigen, 1999) it was then possible to design

overall sampling schemes, given the variogram. It was clear how these automatically balanced the division of effort between variogram estimation and the more spatially regular sampling aimed primarily at ensuring that no site is too far from its nearest neighbouring points for prediction. Since in practice we do not know the variogram at the start of this procedure we proposed a Bayesian framework in which we design the sampling scheme to minimize an error variance over a set of plausible variograms. This offers a framework within which ancillary data or knowledge of variation of the same property in similar conditions could be used to aid sample design.

3. **How can the uncertainty about the specifications of a final sample grid, given a set of observations and their experimental variogram, be quantified, and can this be built into an adaptive sampling scheme for any variable of unknown variability?**
Here we developed a fully adaptive sampling scheme. At the end of each phase of sampling we can compute, in a Bayesian framework, a probability distribution for the interval of a sample grid which will allow us to map the variable of interest to a specified precision. We may then consider the number of additional samples that are needed to complete a survey for geostatistical mapping, such that the target precision is met or exceeded with some probability. We may select a suitably conservative confidence level (e.g. 95%) and compare the number with what we would need if the best estimate of the variogram from our existing data were correct. On the basis of this we can decide whether it is worth collecting further data in the hope that we can then confidently proceed to map from a smaller sample, or whether further reductions in uncertainty about the variogram are likely to be significant. If further samples are to be selected for variogram estimation then the spatial simulated annealing method is used to select the most informative locations to sample.

   We are also able to plan the final survey allowing for the fact that the field of interest is already sampled at many sites, so that an entirely regular grid is not needed in order to achieve a target maximum variance of the estimation error. We demonstrated the utility of these methods in simulation studies.

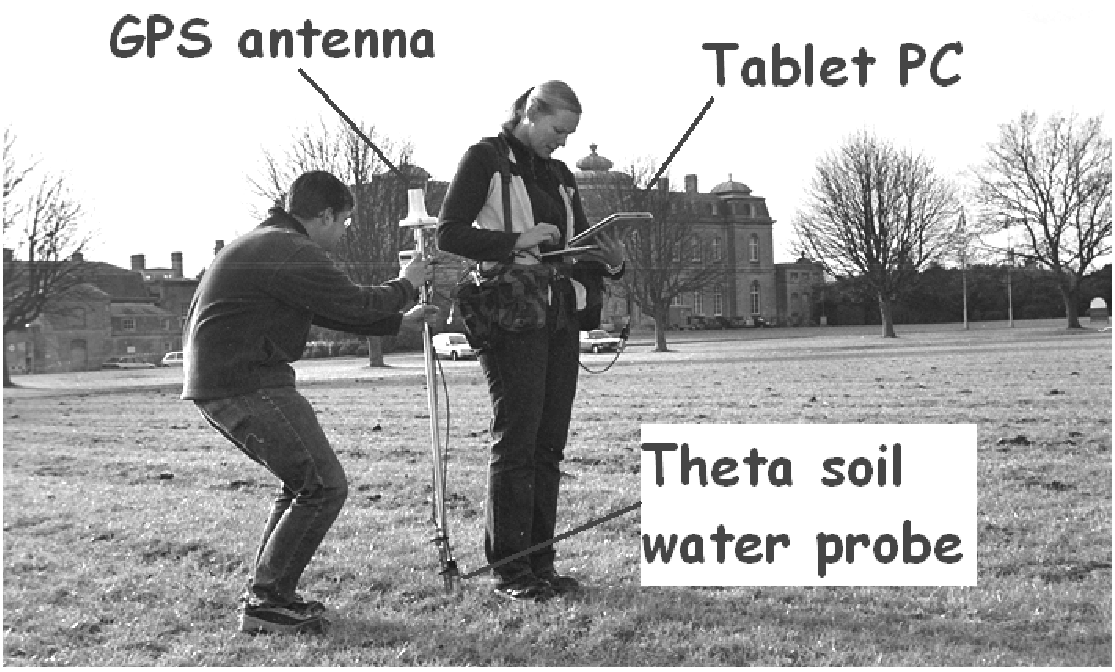
4. **Can these schemes be incorporated into a working field system?**
All the work reported above is statistical research, supported by simulation studies. However, we also wished to show that the approach was feasible in practice. To this end a field system was built. The key components are a Tablet PC which can run the

algorithms, an RTK GPS system for determining position, the antenna of which is mounted on a staff bearing a Delta-T theta probe that measures soil water content. The probe and the GPS are both connected to the PC so measurements and positions can be recorded. The PC also runs an algorithm to direct the user to a sample site selected by one of the methods described above.

Once the system was designed and built we used it in a reconnaissance survey of soil water content in a field as described under (3) above. We specified a target error variance for predictions before the survey, and a total of 75 points were sampled in four phases before it was decided that the uncertainty in the final sample grid could not be reduced further. The final survey could be completed either by adding 15 more points in a regular grid edited to remove redundant points, or by adding 4 more points selected by spatial simulated annealing. We noted that the target error variance was relatively large, and that it would have been easy to over-sample this field without the adaptive approach in force.

The figure below shows the field system in action. The results of this trial survey are shown in the full report.

Figure 0.1. The field system for adaptive sampling for geostatistical mapping of soil water content.



Implications

As we have noted this project was funded primarily by the BBSRC as a piece of research into the basic science of spatial sampling. It was thought to be of strategic relevance to HGCA, rather than generating immediate solutions to Levy-payers' problems. We would suggest that the strategic implications of this work are as follows.

1. We have shown that there is scope to make the sampling of spatial variables less hit-and-miss through the methods that we have developed. The adaptive sampling scheme ensures that, even when we have no prior knowledge about the variability of a property, we end up with a sampling scheme that reflects the spatial scales at which it varies. This can be done without any prior knowledge, although efficiencies may be achieved if we do have some prior information.

2. This approach is most suited to systems where soil properties can be measured in situ, and with increasing interest in development of sensor systems for soil and crop properties this work positions the industry to ensure that such sensors are deployed most efficiently.

3. Despite (2), it is also possible that we could apply these methods to other variables where the cost of soil analysis is large, and the soil property is stable so that delays in time between the phases do not create problems. For example, Oliver & Frogbrook (2004) suggested that some variables, which are stable, might be measured in a single survey which would yield information of long-term value to the farmer. This might be information on physical properties of the soil, such as particle size distribution. These could be mapped adaptively in a number of phases, and given the costs of laboratory determination this might be worthwhile.

4. There is scope to use some of the spatial methods used and developed in this project to study simpler sampling problems in agriculture. We have not discussed this in detail in the main report, since it was not part of the original project proposal. However, the spatial simulated annealing scheme was used to consider how best to form a bulk (composite) sample from which to estimate the mean value of a property in a single field. We specified an average variogram for phosphorous concentration in topsoils of agricultural fields in the Netherlands, as presented by Brus et al (1999). Using this we found a sample design for eight points to sample a square field (right-

hand picture in Figure 0.2) with minimum error variance for the sample value as an estimate of field mean. This design is likely to be relatively insensitive to variogram parameters *unless* we incorporate other factors into the function that we minimize, such as the total distance walked to complete the sample.

We then computed the expected error variances for the actual variograms for each of the fields studied by Brus et al (1999). We plot the cumulative distribution function of the advantages of the optimized scheme relative to the traditional "W" sampling pattern in Figure 0.3. This shows that for over half the fields there is an advantage of over 10% in the precision of the results, and that the optimized system may be as much as 20% superior to the "W". This suggests that there could be advantages in sampling schemes designed by spatial simulated annealing, even for simple sampling problems. Such software would be relatively easy to implement in existing farm management packages, using digitized field boundaries that many farmers already hold for management purposes.

Figure 0.2. A traditional "W" and an optimized sampling scheme for forming a bulk sample of a square field.

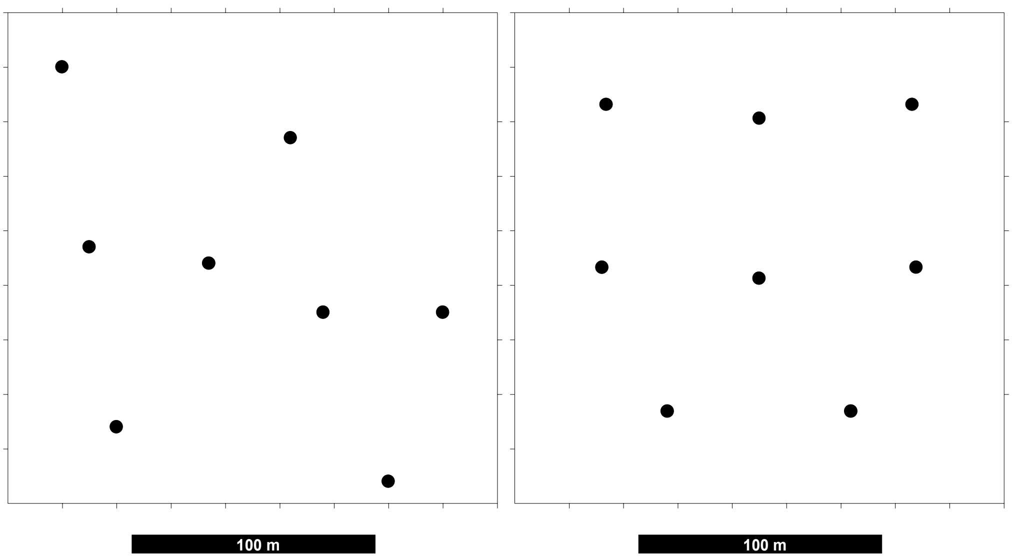
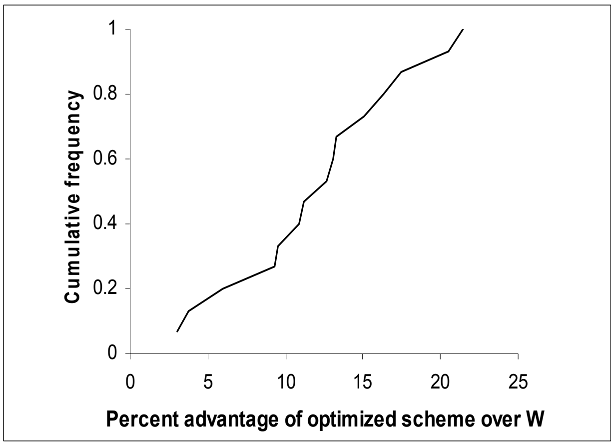


Figure 0.3. Cumulative frequency plot of the % advantage of the optimized sampling scheme over the "W" scheme with respect to the error variance of field average estimates given the 16 variograms presented by Brus et al (1999).



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Lark, R.M., Wheeler, H.C., Bradley, R.I., Mayr, T.R., Dampney, P.M.R. (2003). Developing a cost-effective procedure for investigating within-field variation of soil conditions. Home-Grown Cereals Authority Research and Development Project Report No, 296. HGCA, London.

King, J.A., Dampney, P.M.R., Lark, R.M., Wheeler, H.C., Bradley, R.I., Mayr, T., Russill, N. (2003). Evaluation of non-intrusive sensors for measuring soil physical properties. Home-Grown Cereals Authority Research and Development Project Report No, 302. HGCA, London.

Oliver, M.A., Frogbrook, Z. (2004). Description of spatial variation in soil to optimize cereal management. Home-Grown Cereals Authority Research and Development Project Report No, 330. HGCA, London.

CHAPTER 1. ESTIMATING VARIOGRAM UNCERTAINTY

We do not report this work in any detail here. Information may be found in the following paper, and we include the abstract.

Marchant, B. P. & Lark, R.M. (2004). Estimating variogram uncertainty. *Mathematical Geology*, vol 36, no. 8 pp. 867–898.

Abstract

The variogram is central to any geostatistical survey, but the precision of a variogram estimated from sample data by the method of moments is unknown. In previous studies theoretical expressions have been derived in order to approximate uncertainty in both estimates of the experimental variogram and fitted variogram models. These expressions rely upon various statistical assumptions about the data and are largely untested. They express variogram uncertainty as functions of the sampling positions and the ergodic variogram.

Extensive simulation tests show that for a Gaussian variable with a known isotropic variogram, the uncertainty of the experimental variogram estimate may be accurately determined via these theoretical expressions. In practice however, the variogram of the variable is unknown and the fitted variogram model must instead be used. For sampling schemes of 100 points or more, this is seen to have only a small effect on the accuracy of the uncertainty estimate.

The theoretical expressions generally overestimate the precision of fitted variogram parameters. The uncertainty of the fitted parameters are seen to be more accurately determined by simulating multiple experimental variograms and fitting variogram models to these.

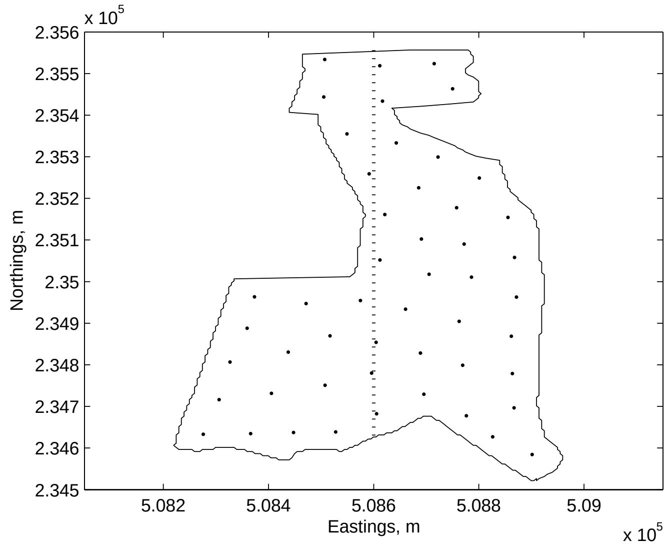
The tests emphasise the importance of distinguishing between the ergodic and non-ergodic variogram. Most studies discussing variogram uncertainty describe the uncertainty associated with estimates to the ergodic variogram. Generally however, it is the non-ergodic variogram which is of interest. For dense sampling schemes, estimates of the non-ergodic variogram are shown to be significantly more precise than estimates of the ergodic variogram. It is important, when designing efficient sampling schemes or fitting variogram models, that the appropriate expression for variogram uncertainty is applied.

CHAPTER 2. OPTIMIZED SAMPLE SCHEMES FOR GEOSTATISTICAL SURVEYS

Introduction

The uncertainty associated with a soil map estimated by ordinary kriging is commonly approximated by the kriging variance. The kriging variance depends upon the sample scheme from which the soil map is estimated and the variogram of the variable being mapped. If the variogram is known exactly then the kriging variance may be calculated prior to sampling. Van Groenigen (1999) has demonstrated when the variogram is known that sample schemes may be optimized prior to sampling to minimize the mean (or maximum) kriging variance over the region where the soil variable is to be mapped. The resulting sample schemes tend to have sample points spread evenly over the region as illustrated in Figure 2.1 which shows a scheme designed to minimize the mean kriging variance for a specified variogram over a number of fields in Silsoe, Bedfordshire, U.K.

Figure 2.1. Test region in Silsoe, Bedfordshire with 50 point sample scheme (dots) designed by minimising the mean kriging variance and 90 point transect (dashes) used for simulation tests.



In reality the variogram is unknown prior to sampling and post sampling it can only be estimated. Sample schemes such as that shown in Figure 2.1 may not be suitable for variogram estimation since no comparisons are made between variable values measured a short distance apart. The uncertainty of the resulting variogram estimate can have an observable effect on the uncertainty of the soil map. Previously Zimmerman and Cressie

(1992) showed that the component of uncertainty in a soil map due to variogram uncertainty may be approximated once the sampled data have been collected. We wish to approximate this component of uncertainty prior to sampling in order to design sample schemes for which the total uncertainty is minimized.

Approximation of the propagation of variogram uncertainty into kriging

In the previous chapter we discussed how the uncertainty in the estimation of a variogram from sampled data may be approximated. Generally, variogram uncertainty is of little interest in itself; the key concern is how this uncertainty propagates into soil maps. We have extended work by Zimmerman and Cressie (1992) to derive an expression for the component of error in soil maps generated by ordinary kriging due to variogram uncertainty. The size of this component of uncertainty depends upon the sample scheme employed and the variogram of the variable but not directly upon the sampled data. Thus if a particular variogram is assumed both the kriging variance and the uncertainty in the soil map due to variogram uncertainty may be calculated prior to sampling. The sum of these quantities is an approximation to the total error in a soil map estimate which we denote σ_T^2.

In this study we focus upon variograms defined by a spherical model:

$$\gamma(h) = c_0 + c_1\left(\frac{3h}{2a} - \frac{1}{2}\left(\frac{h}{a}\right)^3\right) \text{for } 0 < h \le a,$$

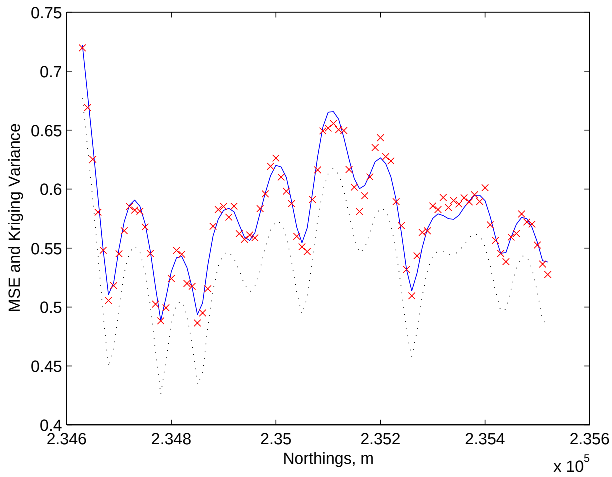
$$\gamma(h) = c_0 + c_1 \text{for } h > a,$$

$$\gamma(0) = 0.$$

Here $\gamma(h)$ denotes the variogram for lag distance h, c_0 denotes the nugget effect, c_1 denotes the sill and a denotes the range of spatial correlation.

Figure 2.2 shows the results of a simulation study, for a particular variogram, where data simulated at the sample points shown in Figure 2.1 are used to make kriged estimates of the variable along the transect also shown in Figure 2.1. The mean squared difference between simulated and kriged values along the transect (continuous line) is greater than the kriging variance (dotted line) due to the effects of variogram uncertainty. The mean squared difference is accurately approximated by our expression for total uncertainty in an ordinary kriging survey σ_T^2; the values of which are denoted by crosses. We find that the expression is accurate for all specified variogram functions provided that the range of spatial correlation is greater than the spacing between sample points.

Figure 2.2. Results of simulation tests for spherical variogram with $c_0 = 0.25$, $c_1 = 0.75$ and $a = 240.0$. The continuous line shows the mean squared difference between kriged estimate and simulated value, the dotted line shows the kriging variance and the crosses show σ_T^2 values.



Optimization of sample schemes for variogram estimation and ordinary kriging

If a particular variogram is assumed, an optimal sample scheme for generating a soil map may be thought to be the configuration of sample points which minimizes the total error in the soil map σ_T^2. Van Groenigen (1999) developed an algorithm known as spatial simulated annealing (SSA) which can select the sample scheme which minimizes such expressions. Figures 2.3-2.6 show sample schemes designed by using the SSA algorithm to minimize the mean value of σ_T^2 across the region shown in Figure 2.1. Different spherical variogram functions are used in each figure.

In Figure 2.3 the position of 50 points are optimized for $c_0 = 0.127$, $c_1 = 1.000$, and $a = 240.0$. The points in the sample scheme which results are spread relatively evenly across the study region apart from three close pairs which aid in the variogram estimation. In Figure 2.4 the range of spatial correlation is reduced to $a = 120.0$. Again, most of the points of the resulting scheme are spread evenly across the region apart from two transects containing three and four closely spaced points. In Figure 2.5 the range is further reduced to $a = 90.0$. In this case the edges of the region are left unsampled. This is an indication that the number of close pairs of points required to estimate the variogram does not leave sufficient points to reduce the kriging variance in all parts of the region. In Figure 2.6 the number of sample points is increased to 125. This leads to a more even distribution of points across the whole region with a number of close pairs for variogram estimation. Further optimizations revealed that the pattern of optimal sample schemes was less sensitive to changes in the nugget and sill parameters.

Figure 2.3. Fifty point sample scheme designed by minimization of σ_T^2 for a spherical variogram with $c_0 = 0.127$, $c_1 = 1.000$ and $a = 240.0$.

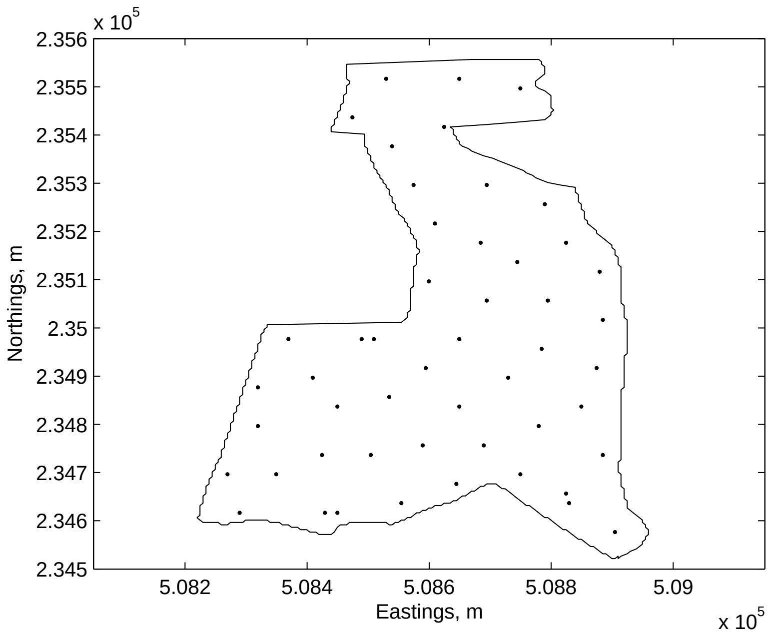


Figure 2.4. Fifty point sample scheme designed by minimization of σ_T^2 for a spherical variogram with $c_0 = 0.127$, $c_1 = 1.000$ and $a = 120.0$.

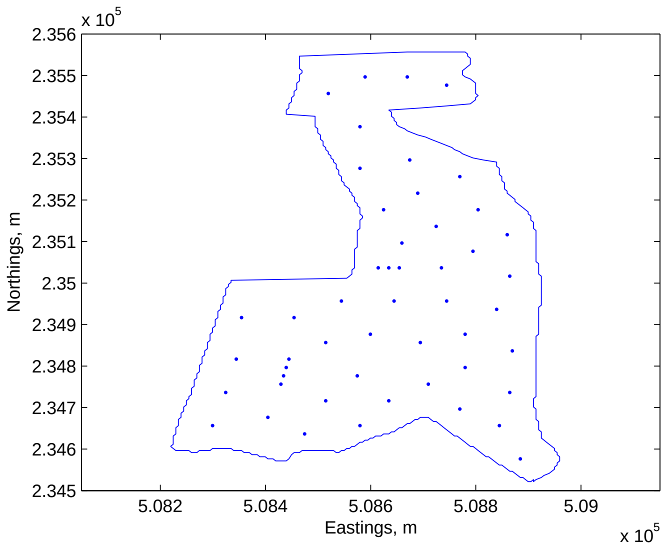


Figure 2.5. Fifty point sample scheme designed by minimization of σ_T^2 for a spherical variogram with $c_0 = 0.127$, $c_1 = 1.000$ and $a = 90.0$.

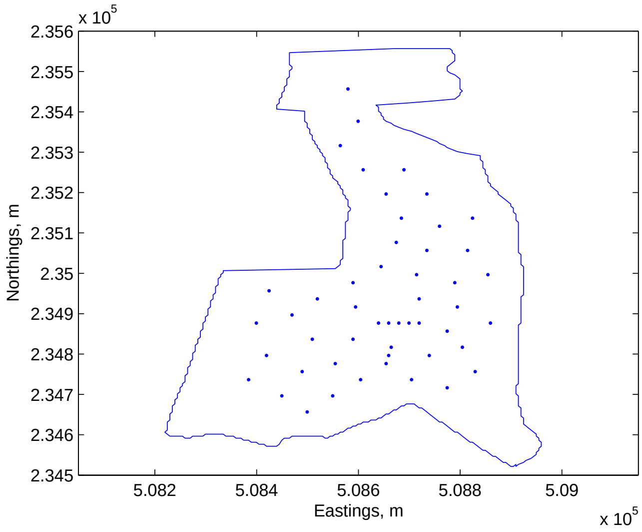
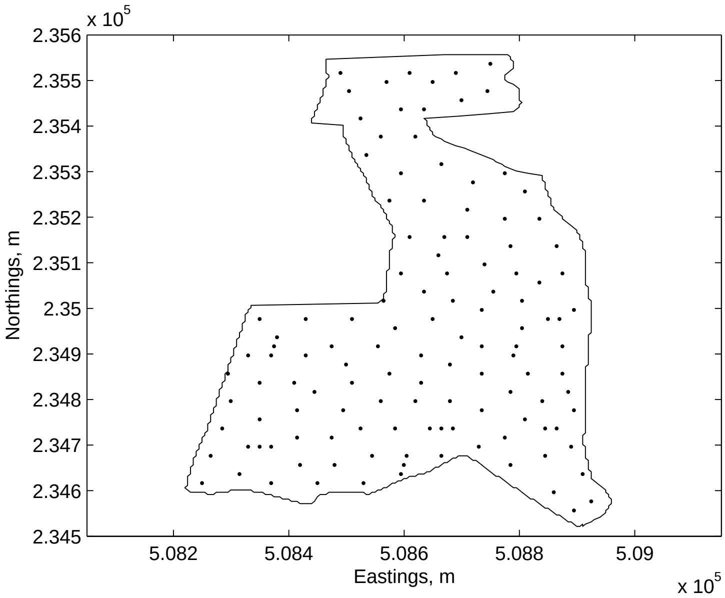


Figure 2.6. One hundred and twenty five point sample scheme designed by minimization of σ_T^2 for a spherical variogram with $c_0 = 0.127$, $c_1 = 1.000$ and $a = 90.0$.



Strategies to account for the sensitivity of optimal sample schemes to variogram parameters

The optimized sample schemes shown in Figures 2.3-2.6 are sensitive to the range of spatial correlation of the variable being measured. The number of close pairs required for variogram estimation increases as the range of spatial correlation decreases. The variogram parameters are unknown prior to sampling so schemes designed with the assumption of a particular variogram may prove to be sub-optimal. This problem may be countered if the sample scheme is optimized over all plausible variogram parameters rather than a single set of variogram parameters. This optimization may be performed within a Bayesian framework.

The Bayesian approach requires the probability density function (pdf) of the variogram parameter vector. The sample scheme is then optimized over all combinations of plausible variogram parameters with a larger weighting given to the most probable parameters. Prior to sampling little will be known about the variogram parameter pdf although previous experience of sampling in similar situations may allow us to place bounds on the plausible values. The pdf may be assumed to be uniform between these bounds and the sample scheme which results will be suitable for all sets of variogram parameters within these bounds. Once some data values have been collected it is possible to update the pdf of variogram parameters using the method described by Pardo-Igúzquiza and Dowd (2003). From the updated pdf the sample scheme may be re-optimized such that it is more suitable for the actual variogram of the variable being mapped. This is an example of an adaptive sampling approach.

Conclusions

Optimal sample schemes from which a variable's variogram may be estimated and unsampled values may be interpolated by ordinary kriging can be designed by the minimization of an approximation to the total error in a soil map. This approximate expression requires the variogram of the variable being mapped as an input. Therefore truly optimal sample schemes may only be designed if the actual variogram is known prior to sampling. In practice this is not the case but if the optimization is carried out within a Bayesian framework it is possible to design schemes which are optimal for generating soil maps based upon the information available about the variogram.

CHAPTER 3. ADAPTIVE SAMPLING FOR RECONNAISSANCE SURVEYS FOR GEOSTATISTICAL MAPPING OF THE SOIL

Introduction

McBratney, Webster and Burgess (1981) suggested a two-phase sampling algorithm to ensure that soil maps have a specified precision. The first phase is a reconnaissance survey, from which the variogram is estimated. The second phase is a regular grid which is suited to ordinary kriging since the points are spread relatively evenly over the region. The required spacing of the points in the regular grid depends upon the threshold placed upon the precision of the soil map and upon the variogram of the variable. McBratney, Webster and Burgess (1981) use the variogram estimated from the reconnaissance survey to estimate the spacing of points required for the regular part of the survey. They select the spacing for which the kriging variance at the centre of the regular grid cells is just less than the precision threshold. This approach does not account for the uncertainty in the estimated variogram.

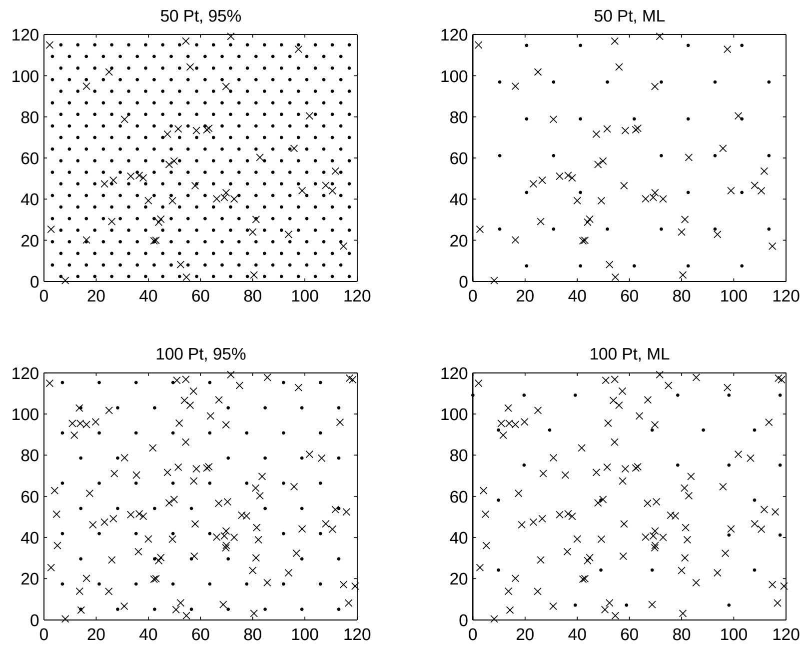
In this chapter we look in more detail at reconnaissance surveys and investigate how the number and position of sample points within them may be optimized and how the spacing of the regular survey may be selected in a manner that accounts for variogram uncertainty. We divide the reconnaissance survey into a number of phases and analyse the data from each phase within the Bayesian framework described in Chapter 2. The probability density function (pdf) of the variogram parameters is updated after each phase of sampling. Each set of variogram parameters corresponds to a particular required sampling interval for the regular survey. Therefore it is possible to calculate a pdf for the sampling interval required by the main survey. This pdf may be used to select a sampling interval for the main survey with a specified degree of confidence that the kriging variance threshold will be satisfied. Also it is possible to assess whether the reconnaissance survey is sufficient or whether the total sampling costs of the survey may be reduced by learning more about the structure of spatial correlation by through another phase of sampling in the reconnaissance survey.

Deciding whether a reconnaissance survey is sufficient

Our Bayesian approach to the assessment of reconnaissance surveys is illustrated in Figures 3.1 and 3.2. In this example the kriging variance threshold has been set at 0.5. The variable being considered has zero mean and a spherical variogram function with $c_0 = 0.05,\ c_1 = 0.8$ and $a = 30.0.$ For this variogram the required sampling interval is 20.25.

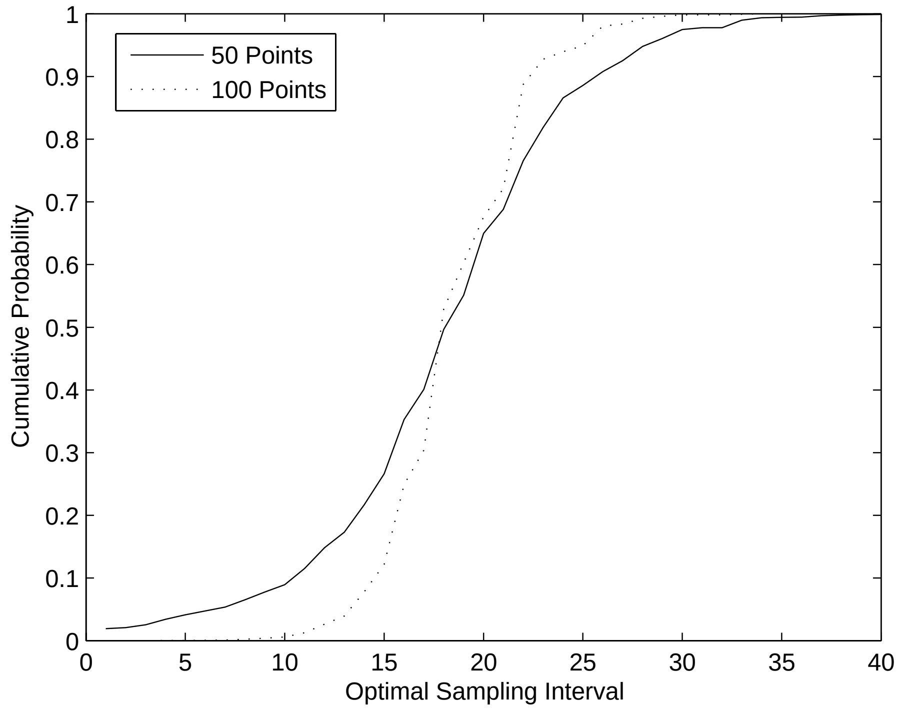
The variable is first simulated on the 50 point random sample schemes marked by crosses in the top left plot on Figure 3.1. Having fitted a variogram to this data the required sampling interval for a triangular main survey may be estimated using the algorithm described by McBratney, Webster and Burgess (1981). After the 50 point reconnaissance survey this approach suggests a main survey with sampling interval equal to 20.5. This is a good estimate of the required sampling interval but the practitioner would be unaware of this. He needs to assess the uncertainty associated with this estimate to decide whether further phases of the reconnaissance survey are required.

Figure 3.1. The position of reconnaissance survey points (crosses) and regular survey points required to satisfy the kriging variance threshold (dots) where the reconnaissance survey consists of 50 (top) or 100 (bottom) points and where the variogram parameters are deduced from the 95% confidence limit (left) or variogram estimate (right).



The continuous line in Figure 3.2 shows the pdf of the required sampling interval calculated by analysing the data from these 50 sample points within the Bayesian framework. A 95% confidence set on this interval stretches from around 5.0m up to 30.0m. According to the pdf a sampling interval of 6.5m would be required to ensure with 95% confidence that the kriging variance threshold is satisfied. A regular survey with this spacing is shown on the top left plot of Figure 3.1. It consists of 388 points. In contrast if the sampling interval suggested by the variogram estimate is applied the regular survey, shown on the top right plot of Figure 3.1 requires a further 39 points. The discrepancy between the sizes of the surveys and the size of the 95% confidence set suggests that the total sampling costs may be reduced via further phases of the reconnaissance survey.

Figure 3.2. The cumulative distribution of required sampling intervals following 50 and 100 point random reconnaissance surveys.



Data values are simulated on a second phase of 50 randomly located points. The complete random survey is denoted by crosses in the lower left plot of Figure 3.1. The variogram estimated from the 100 point reconnaissance survey corresponds to a sampling interval of 19.5. The pdf following the 100 point survey (Figure 3.2, dotted line) says that the lower 95% confidence limit on the required sampling interval is 13.5. A triangular survey with this spacing requires 76 points to cover the region. In comparison if the optimal sampling interval is selected using only the estimated variogram then 29 points are required. Since the saving on sampling with the estimated sampling interval rather than the 95% confidence limit is relatively small and the pdf after 100 points shows that the sampling interval is known with relative certainty we would be unwilling to allocate more sampling effort to the reconnaissance survey.

Selecting sampling locations in a reconnaissance survey

The above example illustrates how the data collected in a reconnaissance survey may be interpreted within a Bayesian framework in order to decide when the required sampling interval is with enough precision to start the main survey. The position of the sample points are selected at random. In this section we describe how the position of sample points may be optimized in order to learn about the required sampling interval with as few sample points as possible. In Chapter 2 we saw that the nature of an optimal sampling scheme for variogram estimation depends on the variogram being estimated. In particular if the range of spatial correlation is reduced more comparisons between close pairs of points are required.

We design reconnaissance surveys to minimize the uncertainty in the estimate of the required sampling interval. This uncertainty depends upon the variogram of the variable being measured. Since we can only estimate this variogram we express the uncertainty in the required sampling interval in terms of the pdf of the variogram parameters. This pdf is re-calculated after each phase of sampling. Thus the sample scheme varies according to the data collected from earlier phases in the reconnaissance survey. We refer to such sample schemes as Bayesian adaptive schemes.

Tables 3.1 and 3.2 compare the performance of Bayesian adaptive schemes with random schemes and regular transects for variables with a short and long range of spatial correlation. Each transect is randomly positioned in the region and contains eight points separated by 10m. The mean value of the lower 95% confidence limit on the required sampling interval and the number of sample points required to complete a triangular survey with this interval, are shown for sample schemes of different sizes. For the short range variable the 95% confidence limit from the Bayesian adaptive scheme is greater than those from the random and transect schemes and the sampling load required to complete the survey is considerably less. In many cases the random and transect schemes do not increase this confidence limit above the minimum permitted value of 1.0.

There is much less difference in the performance of the schemes for the long range variable although the transect scheme has slightly greater confidence limits than the Bayesian adaptive scheme which are in turn slightly greater than those from random schemes. A relatively small number of points are required to survey this variable – a fact which the Bayesian adaptive scheme is able to identify, thus avoiding over-sampling.

Table 3.1. Mean values of 95% confidence limit on required sampling interval and number of points required to complete a survey with this sample interval for different sampling methods on the short range variable.

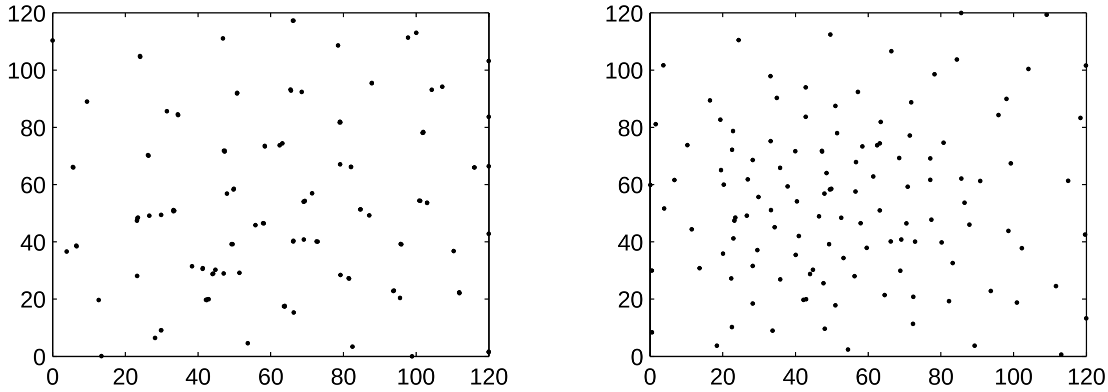
	Sample Size				
Method	28	44	60	76	92
Bayesian Adaptive	1.21m 11500	2.23m 3433	2.89m 2040	3.01m 1903	3.28m 1612
Transects	1.00m 16819	1.02m 16116	1.05m 15180	1.07m 14690	1.10m 11658
Random	1.00m 16819	1.04m 15544	1.04m 15544	1.02m 16116	1.04m 15544

Table 3.2. Mean values of 95% confidence limit on required sampling interval and number of points required to complete a survey with this sample interval for different sampling methods on the long range variable.

	Sample Size				
Method	28	44	60	76	92
Bayesian Adaptive	7.19m 350	11.28m 149	13.18m 110	13.74m 104	14.19m 90
Transects	8.03m 279	11.87m 132	14.27m 90	14.89m 90	15.33m 85
Random	1.94m 4500	3.88m 1134	8.36m 255	11.82m 132	13.92m 95

Figure 3.3 shows typical Bayesian adaptive sample schemes for the long and short range variables. The short range scheme contains a high proportion of close pairs whereas the points in the long range scheme are more evenly spread across the region. Thus the Bayesian adaptive method leads to sample schemes which are specifically suited for the variable being measured and it is effective at estimating the required spacing of the main survey for different variogram functions.

Figure 3.3. Examples of 124 point reconnaissance surveys selected by the Bayesian adaptive algorithm for the short range (left) and long range (right) variables.



Conclusions

The optimal sample scheme for a geostatistical survey depends upon properties of the variable being measured such as its structure of spatial correlation. Efficient sampling can occur if these properties are investigated in a reconnaissance survey before carrying out a main survey. The optimal size and pattern of this reconnaissance survey also depends upon the variogram. If the reconnaissance survey is split into phases the data may be analysed after each phase within a Bayesian framework to allow the design of subsequent phases to be specifically suited to the variable being measured. The Bayesian framework also allows a practitioner to assess how much sampling effort should be allocated to the reconnaissance survey and how much to the main survey.

CHAPTER 4. A FIELD SYSTEM FOR ADAPTIVE SAMPLING

Introduction

The Bayesian adaptive sampling algorithm described in Chapter 3 was implemented in the field system shown in Figure 4.1. The algorithm, running on a hand held computer, selected the sample points required for each phase of the reconnaissance survey. The computer was connected to a global positioning system, the antenna of which was positioned on top of the sensor (in this case a Theta moisture probe). Further software running on the computer aided the practitioner in locating each sample point. The sensor measurement was then entered via the keyboard and the location of the measurement was automatically logged from the GPS.

Figure 4.1. A field system for adaptive sampling

Case Study

The adaptive sampling system was tested in a survey of soil moisture content over a 90×60m field in Silsoe, Bedfordshire, UK. A threshold of $21\%^2$ on the kriging variance was selected. An initial sampling phase of 30 points was designed within the Bayesian framework. A uniform distribution of variogram parameters within the bounds described by Table 4.1 was assumed. Subsequent sampling phases were designed using the Bayesian adaptive algorithm and the updated pdf of variogram parameters.

Table 4.1. Initial bounds placed on variogram parameters in the Silsoe Case Study.

Parameter	Min	Max
c_0	$0.000\%^2$	$15.000\%^2$
c_1	$0.001\%^2$	$30.000\%^2$
a	6.000 m	100.000 m

Results

The Bayesian adaptive algorithm designed a 79 point sample scheme with 95% confidence that the kriging variance threshold was satisfied at each point in the field. The reconnaissance survey consisted of four phases of sampling. Figure 4.2 shows the position of sample points in each of these phases. Table 4.2 shows the lower 95% confidence limit on the optimal sampling interval after each phase and the number of points required to complete a regular survey with this sampling interval. For comparison the sampling interval corresponding to the estimated variogram and the number of regular points required to sample with this interval are also listed. The pdfs of the optimal sampling interval after each phase of sampling are shown in Figure 4.3.

Figure 4.2. The position of points within each phase of the reconnaissance survey for the Silsoe Case Study. The dots denote existing points and the crosses denote points in that phase.

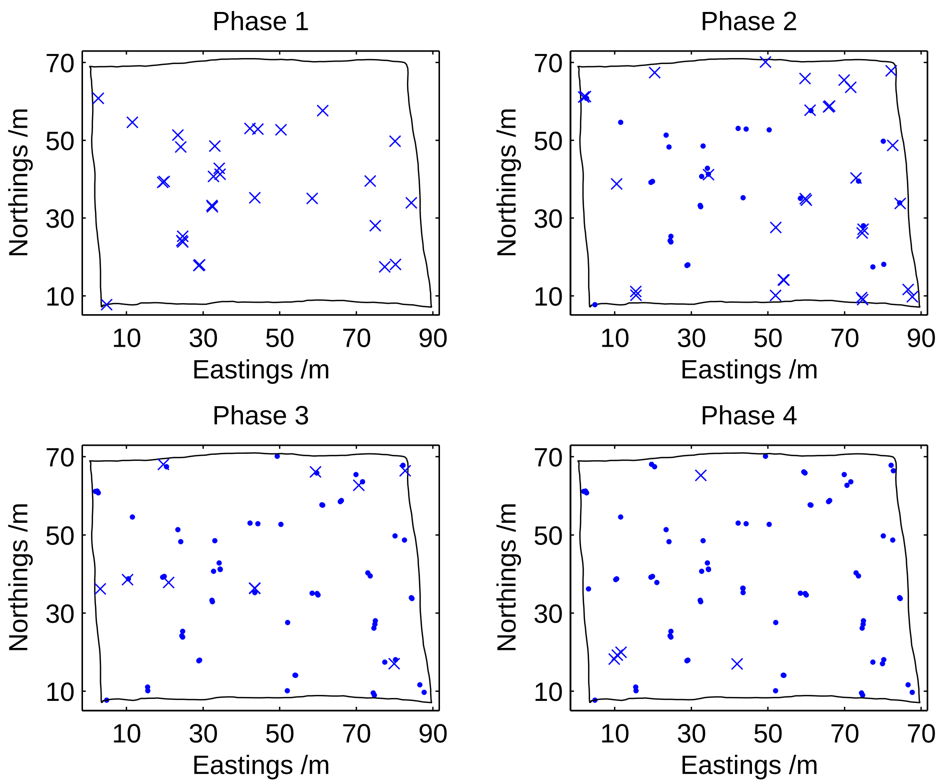


Table 4.2. Results from the Silsoe case study.

N_r	I^{95} \m	N_1^{95}	N_2^{95}	I^e \m	N_1^e	N_2^e
30	6.40	172	169	15.60	55	53
60	12.75	97	94	100.00	60	60
70	16.00	92	90	66.50	71	71
75	17.75	93	90	75.60	76	76

N_r is the number of reconnaissance survey points.

I^{95} is the 95% confidence limit on the required sampling interval.

N_1^{95} is the number of points required to complete survey with interval I^{95}.

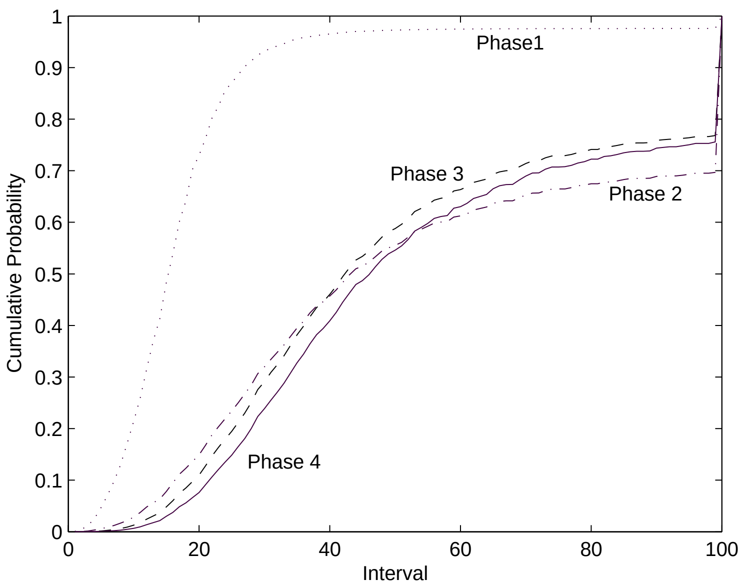
N_2^{95} is the number of points required to complete survey with interval I^{95} once redundant points are removed.

I^e is the estimate of the required sampling interval.

N_1^e is the number of points required to complete survey with interval I^e.

After the first phase of 30 points, Table 4.2 shows that a regular survey containing a further 139 points would be required to sample the moisture content with 95% confidence of satisfying the kriging variance threshold. In comparison a further 23 points would be required if the variogram estimated from this phase was assumed to be correct. The difference in size of these two regular sample schemes suggests that savings may be made by learning more about the variogram. Therefore a second 30 point phase of the reconnaissance survey was completed. Figure 4.3 illustrates how the optimal sampling interval was known with more certainty after this phase.

Figure 4.3. The pdf of the required sampling interval after each phase of the Silsoe Case Study reconnaissance survey.



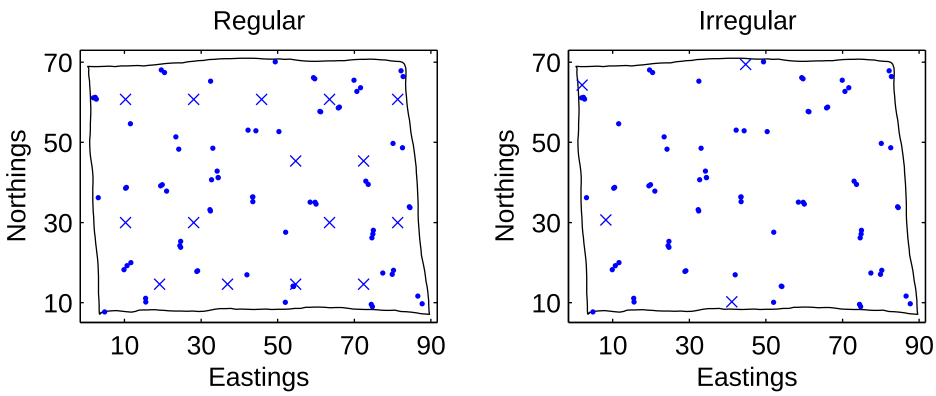
A further 34 regular points were required to complete a regular survey with 95% confidence that the kriging variance threshold will be satisfied. In comparison no further points were required if the estimated variogram was assumed to be correct. After a third reconnaissance phase consisting of 10 points, a further 20 points were required to complete the survey with 95% confidence that the kriging variance threshold would be satisfied. The estimated variogram required 1 further point. After a fourth phase of 5 points, a further 15 points were required to complete the survey with 95% confidence that the kriging variance threshold would be satisfied. Again the estimated variogram required 1 further point. At this point the reconnaissance survey was halted as little saving could be made by learning more about the variogram. The pdfs after the second, third and fourth phases show that there is a probability of approximately 0.3 that the kriging variance threshold will be satisfied for regular grids with spacing of 100m. This is an indication that the selected threshold is fairly lenient and may be greater than the total variance of the moisture content across the field. For this reason a relatively small number of points are required to complete the survey. A non adaptive sampling scheme would not have recognised this, possibly leading to over-sampling of the region.

The left hand plot in Figure 4.4 shows the 75 point reconnaissance survey and the 15 points on the regular grid suggested by the adaptive algorithm. The sample size may be reduced further by permitting irregularly positioned points in the main survey. In the right hand plot of Figure 4.4, the 15 regular points have been replaced by four irregularly positioned points. The number and position of these points were chosen using an iterative spatial simulated annealing algorithm designed to ensure that the kriging variance threshold was satisfied everywhere in the field. This algorithm used the most probable of the sets of variogram parameters which corresponded to a sampling interval equal to the lower 95% confidence limit. Initially a SSA algorithm was used to minimize the maximum kriging variance within the region with 15 points in addition to the reconnaissance survey points (van Groenigen, 1999). If the maximum kriging variance threshold was satisfied one point was removed and the SSA algorithm was repeated. This process continued until there were insufficient points to satisfy the kriging variance threshold. The four points shown on the right hand plot of Figure 4.4 represent the position of the smallest number of points able to satisfy the kriging variance threshold.

Conclusions

The Bayesian adaptive algorithm for reconnaissance surveys was successfully implemented into a field system. This system was used to design a survey to map soil moisture content over a field with 95% confidence that a threshold placed on the kriging variance was satisfied everywhere in the field. The survey is specifically suited to the variable being measured and the threshold placed on it; the relatively small number of points required indicates that the threshold may have been almost as large as the total variance of the moisture content.

Figure 4.4. The suggested sampling positions for the complete survey of moisture content at Silsoe. The dots denote the reconnaissance survey. The crosses in the left hand plot denote the regular sample scheme with redundant points removed and the crosses in the right hand plot denote the irregular survey which ensures that the kriging variance threshold is not exceeded at anywhere in the field.



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